

## **Determination of Elastic modulus of thin materials by Speckle Interferometry**

**Koung-Suk KIM<sup>1</sup>, Ho-Sup CHANG<sup>2</sup>, Sung-Wook JUNG<sup>2</sup>, Tae-Ho CHOI<sup>2</sup>,**

**Woo-Jin KIM<sup>2</sup>, Jae-Gun SONG<sup>2</sup>, and Akhter NASEEM<sup>1</sup>**

**<sup>1</sup>Department of Mechanical Design Engineering, Chosun University,**

**<sup>2</sup>Laser Application Research Centre, Chosun University,**

**375 Seoseok-Dong, Dong-Gu, Gwangju, 501-759, South Korea**

**Tel: +82-10-5120-5016, Fax: +82-62-230-7838**

**E-mail: naseemalig@gmail.com**

### **Abstract**

The paper proposes a sonic resonance test for an elastic modulus measurement which is based on speckle pattern interferometry (SPI). Previous measurement technique of elastic constant has the limitation of application for thin film or polymer material because contact to specimen affects the result. Speckle pattern interferometry (SPI) has been developed as a non-contact optical measurement technique which can visualize resonance vibration mode shapes with whole-field. The maximum vibration amplitude at each vibration mode shape is a clue to find the resonance frequencies. The dynamic elastic constant of test material can be easily estimated from vibration of a beam equation using the measured resonance frequencies. The proposed technique is able to give high accurate elastic modulus of materials through a simple experiment set up and analysis. The result also compared to the theoretical result. In this paper, the basic principles of the technique are described briefly.

**Key Words:** TA-ESPI (time-average speckle pattern interferometry), Non destructive evaluation, Speckle Interferometry, Resonance frequency, Sonic resonance test etc.

### **1. Introduction**

Materials science plays a pivotal role in determining and improving economic performance and the quality of life. Naturally, application of materials is the ultimate goal, but this needs to be built on a firm theoretical basis so that improvements can be made more efficiently and reliably. Greater emphasis needs to be placed on the understanding of mechanical properties of materials and product development. Therefore, it is important to determine the mechanical properties of the thin materials<sup>[1, 2]</sup>. Various testing techniques have been investigated to determine the properties of thin film for example Bulge test, micro tensile test,<sup>[3, 4]</sup> etc. Each have some strong and weak points with respect to specimen preparation procedure of experiment analysis. There are so many techniques have been developed for measuring mechanical properties of thin films materials. Kisoo Knag et al<sup>[5]</sup> used the optical



method to measure the resonance frequency of a cantilever beam to determine its elastic modulus.

This paper proposes a new sonic resonance test for elastic modulus measurement which is based on vibration of a beam at resonance frequency and time-average electronic speckle pattern interferometry (TA-ESPI) which is a kind of laser speckle interferometry with the advantages of non-destructive, non-contact, high resolution and real-time measurement technique of deformation for structures subjected to the various kind of loadings. We used the principle that one can determine the elastic properties of a material by means of its vibration behavior which use the beam equation theory to link with the specimen natural frequency of vibration. In principle, a harmonically vibrating object has the maximum surface displacement at a resonance frequency. The amplitude of the object is directly proportional to TA-ESPI fringe order. The number of the TA-ESPI fringe order is a clue to find the resonance frequency at each vibration mode shape.

## 2. Time Average ESPI

The TA-ESPI has been widely used as the extraction method of vibration amplitude and by means of fringe irradiance measurement from recent decades. For vibration pattern measurement, the interferometer is operated in time average mode. In this mode the images are being recorded and added together while the object is vibrating. The experimental set-up for time-average ESPI is shown in fig-1. One beam of a split Nd: YAG laser light is expanded and illuminated to the object surface and other beam is expanded and passes through ND-filter, which results in the speckled reference beam. The light dispersed by the object and the reference beam are directed to the CCD camera where the beams are interfered.

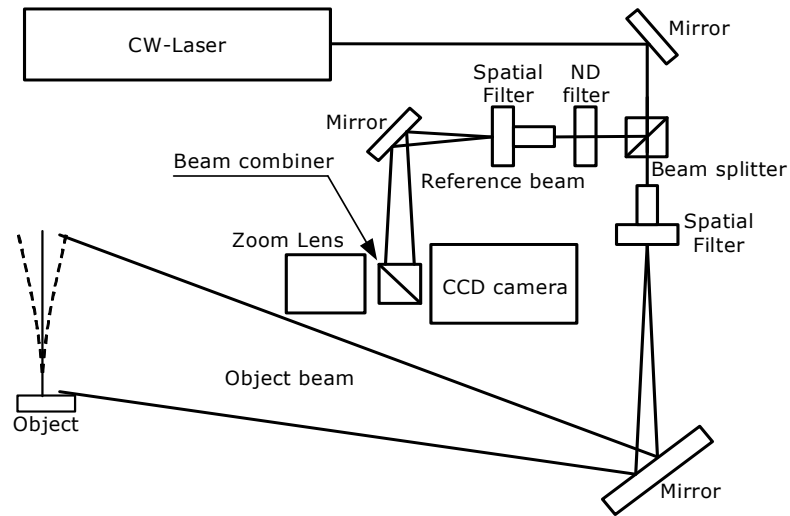


Fig. 1 Out-of-plane displacement sensitive ESPI interferometer

The approach to mathematical description of time-average ESPI is similar to that of the double-exposure variety <sup>[6, 7]</sup>. Discussion is confined to the case where the time varying displacement is along z-axis, which, in the holographic arrangement used, is along the line of sight between object and observer. The displacement is a periodic function of time, and the displacement is simplified if the displacement is allowed to vary along with x and time, that is

$a(t)$  ( $=a_0 \sin \omega t$ ). If  $\Phi(x, y)$  is the resulting phase distribution, then the object complex amplitude is given by

$$U_t(x, y) = A(x, y) e^{i \left[ \phi(x, y) + \frac{4\pi}{\lambda} a(t) \right]} \quad (1)$$

The time-average hologram is recorded with this object beam and the usual reference beam for time  $T$  that is larger than several periods of motion. The reconstructed object wave has complex amplitude that is proportional to the time average of the  $U_t(x, y)$  over the time  $T$ , which is

$$U_{TA}(x, y) = A(x, y) \frac{1}{T} \int_0^T e^{i \left[ \phi(x, y) + \frac{4\pi}{\lambda} a(t) \right]} dt \quad (2)$$

$$U_{TA}(x, y) = A(x, y) e^{i\phi(x, y)} J_0 \left[ \frac{4\pi}{\lambda} a_0 \right] \quad (3)$$

where  $J_0$  is the zero order Bessel Function of the first kind. The irradiance is calculated UTA. UTA\*, which yields,

$$I_{TA}(x, y) = A^2(x, y) J_0^2 \left[ \frac{4\pi}{\lambda} a_0 \right] \quad (4)$$

In this case, the image has superimposed on it a system of fringes that correspond to the minima of the square of the zero order Bessel function. These minima are not equally spaced, and they should not be interpreted like other fringes. The first order has much larger amplitude than subsequent orders, which is what makes the nodal are the brightest in a time-average interferogram<sup>[8]</sup>.

## 2.1 Determination of Elastic modulus.

The natural frequencies of vibration can be obtained by solving the vibration of a beam equation with one end fixed and other end free. Assuming that beam thickness and cross-sectional are is uniform through out its length, the  $n$ th natural frequency of vibration can be calculated by the following equation<sup>[9, 10]</sup>.

$$f_n = \frac{(k_n L)^2}{2\pi L^2} \sqrt{\frac{EI}{\rho A}} \quad (5)$$

where  $n=1,2,3,4,\dots$  and the number  $k_n$  depends upon boundary conditions of the problem,  $L$  is the length of the beam,  $E$  is the elastic modulus,  $I$  stands for moment of inertia and  $\rho$  is the mass density of the beam. Therefore, Elastic modulus can be determined by using equation (5), as

$$E = \frac{4\pi^2 \rho A L^4 f_n^2}{(k_n L)^4 I} \quad (6)$$

### 3. Experimental set-up and specimens

The experimental set is shown in fig.3. It consists of Nd: YAG laser system ( $\lambda = 532\text{nm}$ ), ESPI sensor, Excitation device, Image processing program. The speaker of excitation device selected considering frequency range, function generator adjusts the excitation frequency.

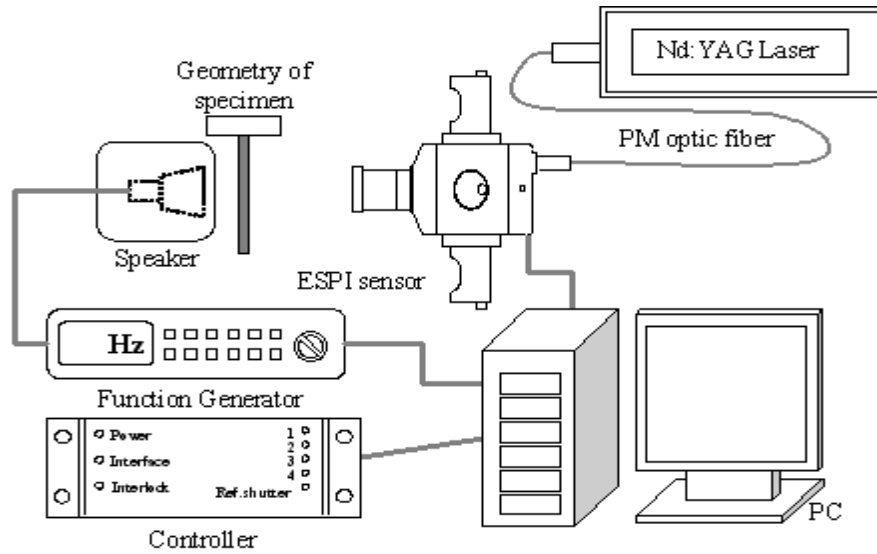


Fig. 3 The schematic of experimental set-up

The distinct frequency characteristic was inspected by using microphone. In present research, the specimens used for experiment were pure Aluminium (99.99%) of lengths, 30mm, 40mm, 50mm, 60mm and 70mm. The thickness and width of each specimen were 50 $\mu\text{m}$  and 5mm respectively. The modulus of elasticity of pure Al (99.99%) is 70Gpa. Properties of materials generally vary depending upon manufacturing process, chemical composition, internal defects, temperature, and dimensions of test specimens, and other factors.<sup>[13]</sup>

### 4. Experiment and results

TA-ESPI corresponding to mode of vibration of object is presented by correlation laser interference pattern. Frequency is impossible to measure by correlation interference pattern technique; therefore for solving this problem, we used vibration method to determine resonant frequency by analyzing resonance mode in real time. The Elastic modulus and mass density of pure Al (99.99%) are 70Gpa and 2710 kg/m<sup>3</sup> respectively. In this experiment, the theoretically calculated resonance frequencies and measured by TA-ESPI are determined and

compared. The comparison of resonance frequency with modes of vibration is shown in fig.4 (a). The variation of elastic moduli with vibration modes is also shown in fig.4 (b).

The elastic moduli have been investigated by using beam equation (6). The L/B ratios and elastic moduli with each modes of vibration are shown in fig.4(c). The average elastic moduli for (L/B = 6, 8) was 64.38GPa and 68.52GPa for 10, 12, 14 L/B ratios. In the case of (L/B=6, 8), the maximum errors were 6.807% and 5.07% respectively and in other cases less than 1.8%. It is also found that error is decreasing with increasing L/B ratio. Experimental result shows that the beam's length should be more than ten times of the width and thickness.

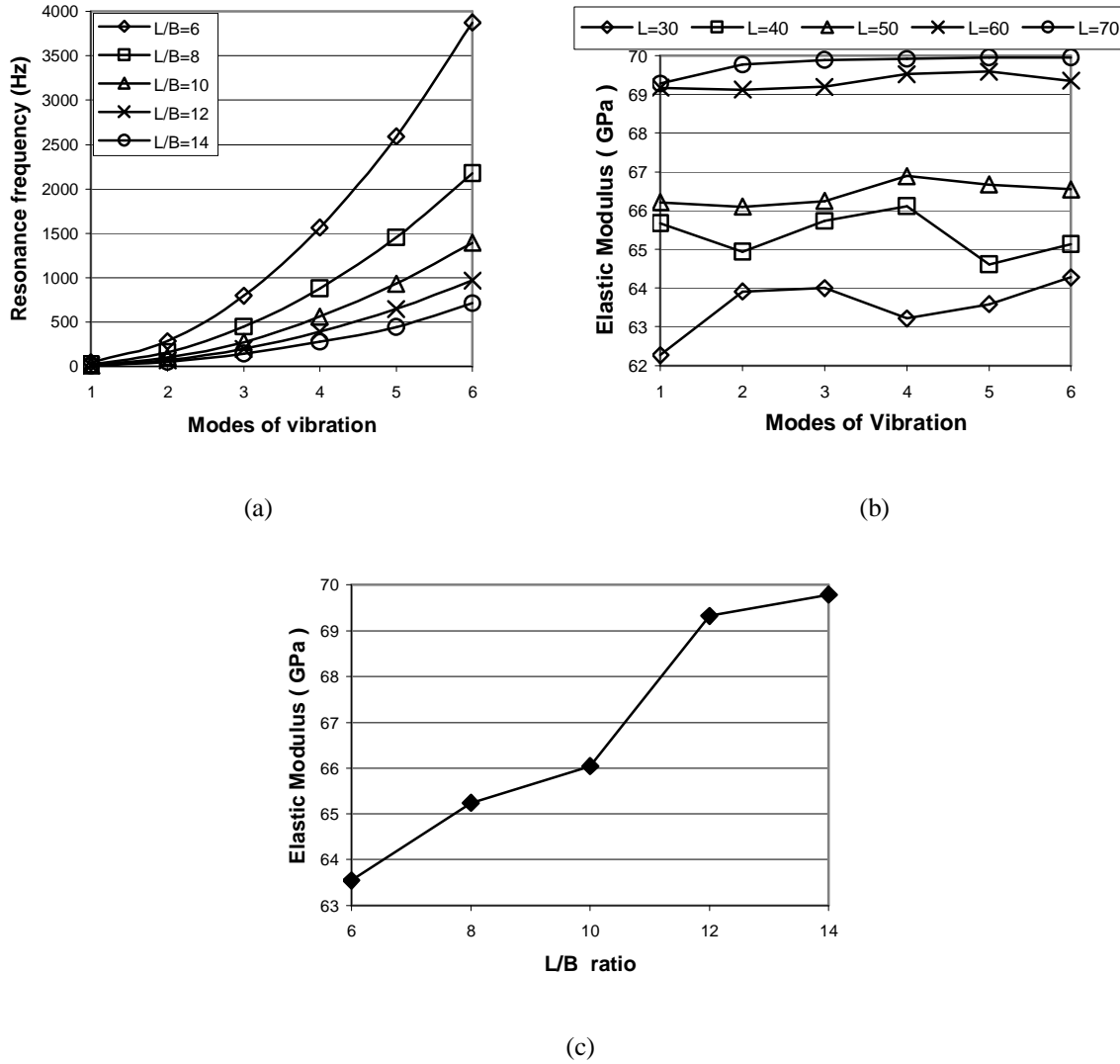


Fig.4. Comparison of resonance frequency, elastic modulus with vibration modes and L/B ratio respectively, (a) is theoretically determined value; (b) and (c) are determined by TA-ESPI result

#### 4. Conclusion

The mechanical properties, namely elastic modulus is a very important parameter to determine and analysis of stress and strain of a material. In this study, the method used time-

averaged electronic speckle pattern interferometry (TA-ESPI) to study and determine elastic modulus and its variation with length and width ratio of vibrating Aluminium beam. The elastic modulus can be easily determined by classical vibrating beam theory once get resonance frequencies from TA-ESPI, although in many cases it is very difficult to determine precisely and conveniently. The time-averaged electronic speckle interferometry (TA-ESPI) approach is very simple; it can be applied as a supplement to the measurement of other elastic modulus. Furthermore, one can also determine the other material properties such as shear modulus and Poisson's ratio by the method of speckle interferometry.

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## References

- [1] Yi, C.H., Bahk, S.M., Kim, H.S., "Dynamic Elastic Constants of Composite Material using Resonance Frequencies, Euler and Timoshenko Beam Equation," Korea Journal of Material Research, Vol. 9, No. 7, pp. 670-674, 1999.
- [2] Tsai H-C, Fang W. Determining the Poisson's ratio of thin film materials using resonant method. Sensor Actuators A 2003; 103:377-83.
- [3] Vlassak JJ, Nix WD. A new bulge test technique for the determination of Young's modulus and Poisson's ratio of thin films. J Mater Res 1992;7(12):3242-9.
- [4] Sharpe Jr. WN, Yuan B, Edwards RL. A new technique for measuring the mechanical properties of thin films. J. Microelectromech Syst 1997; 6(3):193-9.
- [5] Kisoo Kang, Koungsuk Kim Hangseo Lee "Evaluation of elastic modulus of cantilever beam by TA-ESPI" Optics and Laser Technology 39 (2007) 449-452.
- [6] Cloud, G.L., Optical Methods of Engineering Analysis, Cambridge University Press, Chapter 21, 1990.
- [7] Inman, D.J., Engineering Vibration, Prentice-Hall Inc., Chapter 9, 1994.
- [8] Robert Jones, Holographic and speckle interferometry, 2<sup>nd</sup> ed. Cambridge University Press, Chapter 2, 1989.
- [9] W. Weaver, JR., S.P. Timoshenko, D.H. Young, Vibration problems in engineering, Fifth Edition, A Wiley-Interscience Publication, Chapter 5
- [10] William T. Thomson, Theory of vibration with applications (4<sup>th</sup> edition), Prentice Hall Inc. chapter 9.